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Dual solutions for unsteady mixed convection flow of MHD micropolar fluid over a stretching/shrinking sheet with non-uniform heat source/sink

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ABSTRACT

The aim of the present study is to investigate the influence of non-uniform heat source/sink, mass transfer and chemical reaction on an unsteady mixed convection boundary layer flow of a magneto-micropolar fluid past a stretching/shrinking sheet in the presence of viscous dissipation and suction/injection. The governing equations of the flow, heat and mass transfer are transformed into system of nonlinear ordinary differential equations by using similarity transformation and then solved numerically using Shooting technique with Matlab Package. The influence of non-dimensional governing parameters on velocity, microrotation, temperature and concentration profiles are discussed and presented with the help of their graphical representations. Also, friction factor, heat and mass transfer rates have been computed and presented through tables. Under some special conditions, present results are compared with the existed results to check the accuracy and validity of the present study. An excellent agreement is observed with the existed results.

1. Introduction

The number of findings in a convective flow and heat transfer over a stretching or shrinking sheet has considerably increased due to its importance in applied sciences. Many researchers have attracted by micropolar fluids because of the equation governing the flow of micropolar fluid involves a microrotation vector and a gyration parameter in addition to the definitive velocity vector field. These parameters plays major role in engineering and allied areas. Micropolar fluid is a subject of microphoric fluid theory. The detailed description and the modeling of micropolar fluids were initially introduced by Eringen [1]. MHD stagnation point flow of a micropolar fluid towards a non-linear stretching sheet was discussed by Hayat et al. [2]. The similar type of study over a shrinking surface was considered by Lok et al. [3]. Papautsky et al. [4] discussed micropolar fluid flow over a rectangular micro channel; they analyzed the momentum and heat transfer behavior of micropolar fluid.

Chen et al. [5] explained the theory and modeling of the micropolar fluid dynamics. Unsteady MHD mixed convection flow of a micropolar fluid in the presence of viscous dissipation and radiation was analyzed by Ibrahim et al. [6]. MHD boundary layer flow of a micropolar fluid over a vertical plate with convective boundary conditions was studied by Aman et al. [7]. Das [8] presented MHD mixed convection flow of a micropolar fluid towards a shrinking vertical sheet by considering slip effects. Zaimi and Ishak [9] illustrated stagnation-point flow and heat transfer behavior of micropolar fluid over a nonlinearly stretching/shrinking sheet. Recently, unsteady mixed convection flow of a micropolar fluid over a permeable shrinking sheet by using finite element analysis was discussed by Gupta et al. [10]. Ashraf and Batool [11] analyzed MHD flow and heat transfer behavior of a micropolar fluid over a stretchable disk. Nazar et al. [12] discussed stagnation-point flow of a micropolar fluid towards a stretching sheet. Mostafa et al. [13] analyzed MHD flow and heat transfer behavior of a micropolar

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fluid in presence of heat source or sink and slip velocity. Jat et al. [14] studied MHD stagnation-point flow of a micropolar fluid by considering heat generation/absorption. Ishak et al. [15] discussed stagnation-point flow of a micropolar fluid over a shrinking sheet. Srinivasacharya and Ram Reddy [16] presented mixed convection flow of a micropolar fluid in the presence of soret and dufour effects.

Homogeneous, heterogeneous reactions in a micropolar fluid flow from a permeable stretching or shrinking sheet in porous medium was analyzed by Shaw et al. [17]. Mixed convection flow of a micropolar fluid over a stretching sheet by considering viscous dissipation effect was discussed by Aziz [18]. Rashidi et al. [19] presented an analytical solution for heat transfer behavior of a micropolar fluid in a porous medium in the presence of radiation. Bhattacharyya et al. [20] discussed influence of thermal radiation on micropolar fluid past a porous shrinking sheet. Melting heat transfer stagnation-point flow of a micropolar fluid towards a stretching/shrinking sheet was discussed by Yacob et al. [21]. Bakr [22] studied influence of chemical reaction on MHD micropolar fluid flow over an oscillatory plate with constant heat source. Kim and Fedorov [23] presented mixed convective radiative MHD micropolar fluid flow past a semi-infinite moving vertical porous plate. Mohan Krishna et al. [24] discussed influence of radiation and chemical reaction of Kuvshinshiki fluid through vertical porous sheet with heat source. Rahman and Sattar [25] analyzed transient convection micropolar fluid flow over a continuously moving vertical porous plate in presence of radiation. Boundary layer flow of a micropolar fluid in porous media was discussed by Raptis [26]. The finite element solution for mixed convective micropolar fluid past a porous stretching sheet was discussed by Bhargava et al. [27]. Desseaux and Kelson [28] analyzed the flow behavior of a micropolar fluid over a stretching sheet. MHD flow and heat transfer of a dusty fluid over a stretching sheet was studied by Gireesha et al. [29]. Hsiao [30] discussed the heat and mass transfer in MHD mixed convection flow of viscoelastic fluid over a stretching sheet with ohmic dissipation. Nanofluid flow by considering multimedia physical features for conjugate mixed convection and radiation was analyzed by Hsiao [31]. Heat and mass transfer of a micropolar fluid flow in presence of radiation over a nonlinearly stretching sheet was illustrated by Hsiao [32]. MHD mixed convection flow of a viscoelastic fluid past a porous wedge was discussed by Hsiao [33].

Present problem is the extension study of Gupta et al. [10]. In this study we analyzed the unsteady mixed convection flow of micropolar fluid past a stretching or shrinking sheet by considering the additional assumptions like Magneticfield, Stretching sheet, injection, diffusion, chemical reaction and the space and temperature dependent heat generation/absorption (non uniform heat source/sink) along with the different numerical technique. We presented dual solutions for suction and injection cases and compared the results of the present study with the existed results of Gupta et al. [10]. Under some special conditions, present results have an excellent agreement with the results of Gupta et al. [10]. This proves the validity of the present results along with the numerical technique we used in this study.

2. Mathematical formulation

Consider an unsteady two-dimensional mixed convection flow of an incompressible boundary layer flow of a magneto-micropolar fluid over a permeable stretching/shrinking sheet. At $t<0$ the fluid is in steady state and at $t\geq 0$, the fluid, heat and mass flows are in unsteady state. We considered a stretching/shrinking velocity $u_w(x,t)=ax/(1-et)$, temperature and concentration of the sheet are respectively $T_w(x,t)=T_\infty+bx/(1-et)$ and $C_w(x,t)=C_\infty+bx/(1-et)$, where a, b and e are constants. The stretching/shrinking sheet is taken along the x-axis and y-axis is normal to it. The magneticfield of strength B (Gireesha et al. [29]) is applied along y-direction. Non-uniform heat source/sink, viscous dissipation along with chemical reaction effects are taken into account. Under the above assumptions, the governing equations of the flow are

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

Momentum equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\mu+S}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{S}{\rho}\frac{\partial N}{\partial y}+g_e\beta(T-T_\infty)+g_e\beta^*(C-C_\infty)-\frac{\sigma B^2}{\rho}u, \tag{2}$$

Angular momentum equation

$$\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{S}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right), \tag{3}$$

Energy equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{(\mu+S)}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2}{\rho c_p}u^2+q''', \tag{4}$$

Species equation

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-k_1(C-C_\infty), \tag{5}$$

With the boundary conditions

$$\left.\begin{aligned}&u=\lambda U_w(x,t),\;\; v=V_w,\;\; N=-\frac{1}{2}\frac{\partial u}{\partial y},\;\; T=T_w(x,t),\;\; C=C_w(x,t),\;\; \text{at } y=0\\ &u\to 0,\;\; N\to 0,\;\; T\to T_\infty,\;\; C\to C_\infty,\;\; \text{as } y\to\infty\end{aligned}\right\} \tag{6}$$

where λ is the stretching/shrinking parameter, $\lambda>0$ for stretching and $\lambda<0$ for shrinking, V_w is the suction/injection parameter, $V_w>0$ for injection and $V_w<0$ for suction, q''' is the non-uniform heat source/sink and is defined as (Gireesha et al. [29])

$$q'''=\left(\frac{kU_w(x,t)}{x\nu}\right)\left(A^*(T_w-T_\infty)f'+B^*(T-T_\infty)\right), \tag{7}$$

where A^* and B^* are parameters of the space and temperature dependent internal heat generation/absorption. The positive and negative values of A^* and B^* represents heat generation and absorption respectively.

We now introduce the following similarity variables to get the similarity transformation for the equations (1)–(5) subject to the boundary conditions (6)

$$\left.\begin{array}{l} \eta=\sqrt{\dfrac{a}{\upsilon(1-et)}}y,\ \ \psi=\sqrt{\dfrac{a\upsilon}{(1-et)}}xf(\eta),\ N=\sqrt[3]{\dfrac{a}{(1-et)}}\dfrac{x}{\sqrt{\upsilon}}h,\\ \theta=\dfrac{T-T_\infty}{T_w-T_\infty},\ \ \phi=\dfrac{C-C_\infty}{C_w-C_\infty},\ \ B=\dfrac{B_0}{\sqrt{1-et}},\ \ k_1=\dfrac{k_0}{(1-et)} \end{array}\right\} \tag{8}$$

where ψ is the stream function, which is defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which identically satisfies continuity equation (1). On applying transformations, equations (2)–(5) reduced to

$$(1+K)f''' + ff'' - \tau\left(f'+\frac{1}{2}\eta f''\right) - f'^2 + Kh' + \delta\theta + \delta_1\phi - Mf' = 0 \tag{9}$$

$$\left(1+\frac{K}{2}\right)h'' + fh' - f'h - \tau\left(\frac{3}{2}h+\frac{1}{2}\eta h'\right) - K\left(2h+f''\right) = 0 \tag{10}$$

$$\theta'' + Pr(f\theta' - f'\theta) + (1+K)PrEcf''^2 - Pr\tau\left(\theta+\frac{1}{2}\eta\theta'\right) + PrMEcf'^2 + A^*f' + B^*\theta = 0 \tag{11}$$

$$\phi'' + Sc(f\phi' - f'\phi) - Sc\tau\left(\phi+\frac{1}{2}\eta\phi'\right) - ScKr\phi = 0 \tag{12}$$

with the transformed boundary conditions

$$\left.\begin{array}{l} f(0)=f_w,\ \ f'(0)=\lambda,\ \ h(0)=-\dfrac{1}{2}f''(0),\ \ \theta(0)=1,\ \ \phi(0)=1,\\ f'(\infty)=0,\ \ h(\infty)=0,\ \ \theta(\infty)=0,\ \ \phi(\infty)=0, \end{array}\right\} \tag{13}$$

where prime denotes the differentiation with respect to η. K is the micropolar parameter, δ is the thermal buoyancy parameter, δ_1 is the concentration buoyancy parameter, τ is the unsteadiness parameter, M is the magneticfield parameter, Pr is the Prandtl number, Ec is the Eckert number, A^*, B^* are the non-uniform heat source/sink parameters, Sc is the Schmidt number, Kr is the chemical reaction parameter and f_w is the suction/injection parameter, $f_w>0$ for suction and $f_w<0$ for injection, these are given by

$$\left.\begin{array}{l} K=\dfrac{S}{\mu},\ \ \delta=\dfrac{Gr_x}{(Re_x)^2},\ \ \delta_1=\dfrac{Gc_x}{(Re_x)^2},\ \ \tau=\dfrac{a}{e},\ \ M=\dfrac{\sigma B_0^2}{a\rho},\ \ Pr=\dfrac{\mu c_p}{k},\\ Ec=\dfrac{U_w^2}{c_p(T_w-T_\infty)},\ \ f_w=-\dfrac{v_0}{\sqrt{a\upsilon}},\ \ Sc=\dfrac{\upsilon}{D_m},\ \ Kr=\dfrac{k_0}{a}, \end{array}\right\} \tag{14}$$

For engineering interest the local skin friction coefficient, Nusselt and Sherwood numbers are given by

$$C_f=\frac{2\tau_w}{\rho U_w^2},\ \ Nu_x=\frac{q_w x}{k(T_w-T_\infty)},\ \ Sh_x=\frac{q_m x}{k(C_w-C_\infty)}, \tag{15}$$

where the wall shear stress τ_w, the heat flux q_w and the mass flux q_m are given by

$$\begin{array}{l} \tau_w=-\left[(\mu+S)\dfrac{\partial u}{\partial y}+NS\right]_{y=0},\ \ q_w=-k\left(\dfrac{\partial T}{\partial y}\right)_{y=0},\\ q_w=-k\left(\dfrac{\partial C}{\partial y}\right)_{y=0}, \end{array} \tag{16}$$

Using the similarity transformation given in (8), we get the following

$$C_fRe_x^{1/2}=-(2+K)f''(0),\ Nu_xRe_x^{-1/2}=-\theta'(0),\ Sh_xRe_x^{-1/2}=-\phi'(0), \tag{17}$$

where $Re_x=U_w x/\upsilon$ is the local Reynolds number.

3. Method of solution

The set of equations (9)–(12) subject to the boundary conditions (13) have been solved numerically using the shooting method. We consider $f=f_1,\ f'=f_2,\ f''=f_3,\ h=f_4,\ h'=f_5,\ \theta=f_6,\ \theta'=f_7,\ \phi=f_8,\ \phi'=f_9$. Equations (9)–(12) are transformed into systems of first order differential equations as follows:

$$\left.\begin{array}{l} f'=f_2\\ f_2'=f_3\\ f_3'=-\dfrac{1}{(1+K)}\left[f_1f_3-\tau\left(f_2+\dfrac{1}{2}\eta f_3\right)-f_2^2+Kf_5+\delta f_6+\delta_1 f_8-Mf_2\right]\\ f_4'=f_5\\ f_5'=-\dfrac{2}{(2+K)}\left[f_1f_5-f_2f_4-\tau\left(\dfrac{3}{2}f_4+\dfrac{1}{2}\eta f_5\right)-K(2f_4+f_3)\right]\\ f_6'=f_7\\ f_7'=-\left[Pr(f_1f_7-f_2f_6)+(1+K)PrEcf_3^2-Pr\tau\left(f_6+\dfrac{1}{2}\eta f_7\right)\right.\\ \qquad\left.+PrMEcf_2^2+A^*f_2+B^*f_6\right]\\ f_8'=f_9\\ f_9'=-\left[Sc(f_1f_9-f_2f_8)-Sc\tau\left(f_8+\dfrac{1}{2}\eta f_9\right)-ScKrf_8\right] \end{array}\right\} \tag{18}$$

Subject to the following initial conditions

$$\left.\begin{array}{l} f_1(0)=f_w,\ \ f_2(0)=\lambda,\ \ f_3(0)=s_1,\ \ f_4(0)=-(1/2)s_1,\\ x_5(0)=s_2,\ \ f_6(0)=1,\ \ f_7(0)=s_3,\ \ f_8(0)=1,\ \ f_9(0)=s_4, \end{array}\right\} \tag{19}$$

In shooting method, we assume the unspecified initial conditions s_1, s_2, s_3 and s_4 in equation (19), equation (18) is then integrated numerically as an initial valued problem to a given terminal point. We can check the accuracy of the assumed missing initial condition, by comparing the calculated value of the different variable at the terminal point with the given value by the existence of the difference in improved values so that the missing initial conditions must be obtained. The calculations are carried out by using the MATLAB programming.

4. Results and discussion

Equations (9)–(12) with the boundary conditions (13) have been solved numerically using Shooting technique. For numerical results we considered the non dimensional parameter values as $K = \tau = M = \delta = \delta_1 = 1$, $Pr = 0.71$, $Ec = 0.75$, $A^* = B^* = 0.1$, $Kr = 0.2$, $\lambda = 1$, $f_w = \pm 1$ and $Sc = 0.2$. These values are kept as constant in entire study except the varied parameters as shown in figures and tables. The results obtained shows the influences of the non dimensional governing parameters, namely magneticfield parameter, micropolar parameter, unsteadiness parameter, non-uniform heat source/sink parameters and the chemical reaction parameter on velocity, microratation, temperature and concentration profiles along with friction factor, Nusselt and Sherwood numbers.

Figs. 1–4 depict the influence of magneticfield parameter on velocity, microrotation, temperature and concentration profiles respectively for both suction and injection cases. It is evident from the Figs. 1 and 2 that an increase in magneticfield parameter depreciates the velocity distribution and enhances the microrotation near the boundary and at $\eta_\infty = 2$ level microrotation takes reverse action and follows the velocity profiles of the flow. But enhancement in magneticfield parameter increases the temperature and concentration distribution as displayed in Figs. 3 and 4. This is due the fact that strengthening the magneticfield causes to develop the opposite force to the flow, is called Lorentz force. This force works opposite to the flow and declines the velocity profiles at the same time it help to increase the thermal and concentration boundary layers. From figures it is observed that influence of magneticfield parameter is more effective in injection case compared with suction case. It is also observed an overshoot in temperature profiles as displayed in Fig. 3. This may happen due to the injection effect. Generally injection allows the fluid to enter into the system and hence the magneticfield parameter effectively increases the thermal boundary layer thickness in injection case.

Figs. 5–7 illustrate the influence of micropolar parameter on velocity, microrotation and temperature profiles respectively for both suction and injection cases. It is observed from the figures that enhancement in micropolar parameter increases the velocity, microrotation and temperature profiles for both suction and injection cases. It is due to the fact that an increase in micropolar

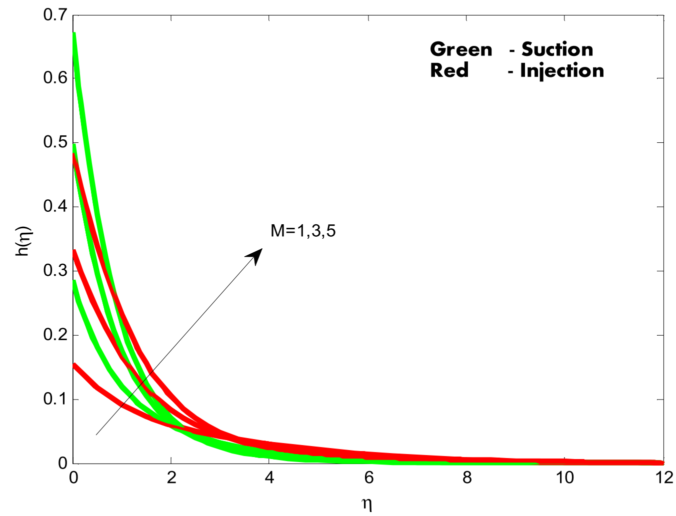

Fig. 2. Microrotation distribution for different values of magneticfield parameter *M*.

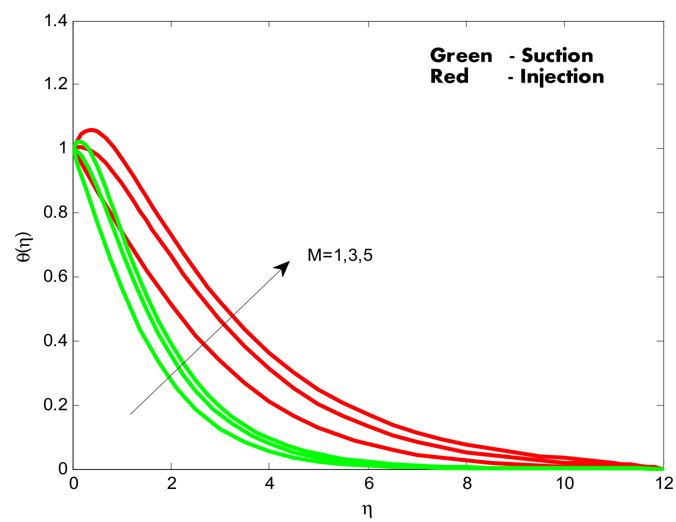

Fig. 3. Temperature distribution for different values of magneticfield parameter *M*.

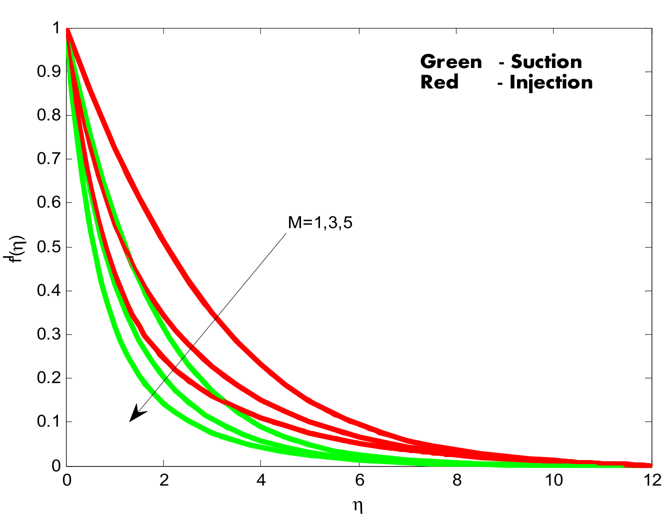

Fig. 1. Velocity distribution for different values of magneticfield parameter *M*.

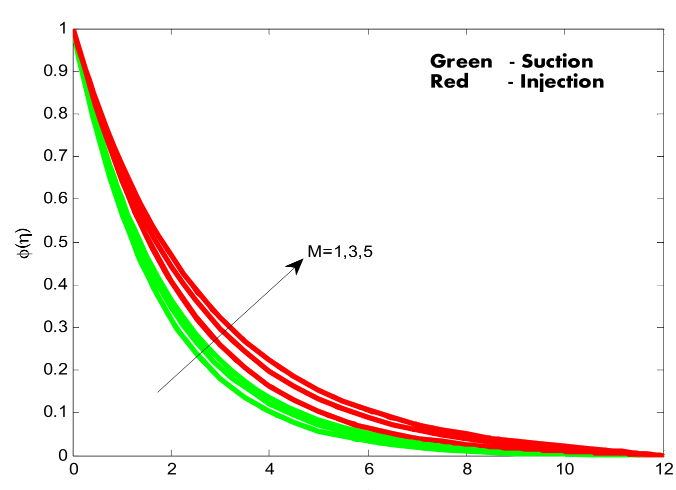

Fig. 4. Concentration distribution for different values of magneticfield parameter *M*.

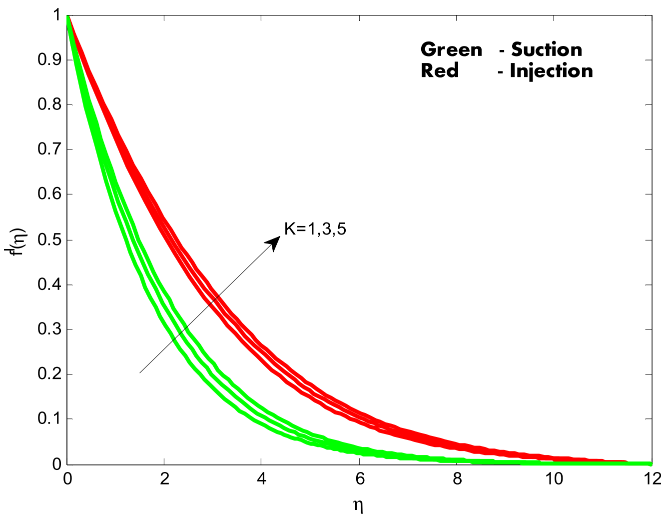

Fig. 5. Velocity distribution for different values of micropolar parameter *K*.

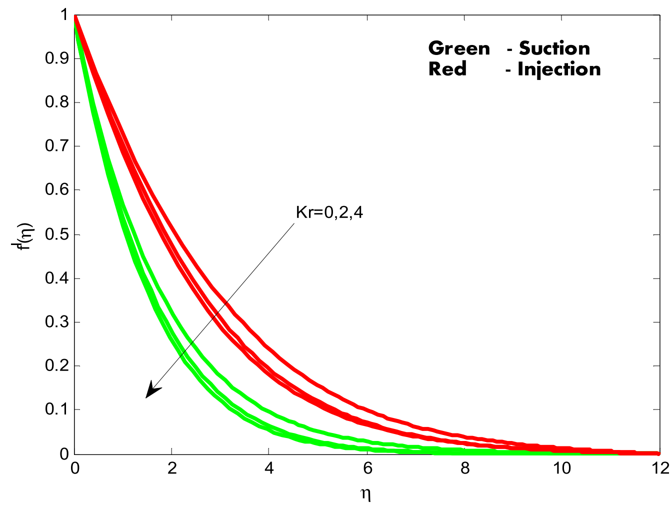

Fig. 8. Velocity distribution for different values of chemical reaction parameter *Kr*.

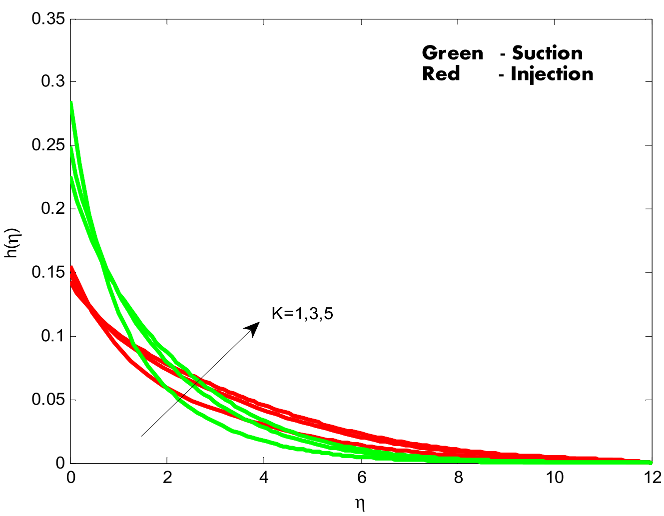

Fig. 6. Microrotation distribution for different values of micropolar parameter *K*.

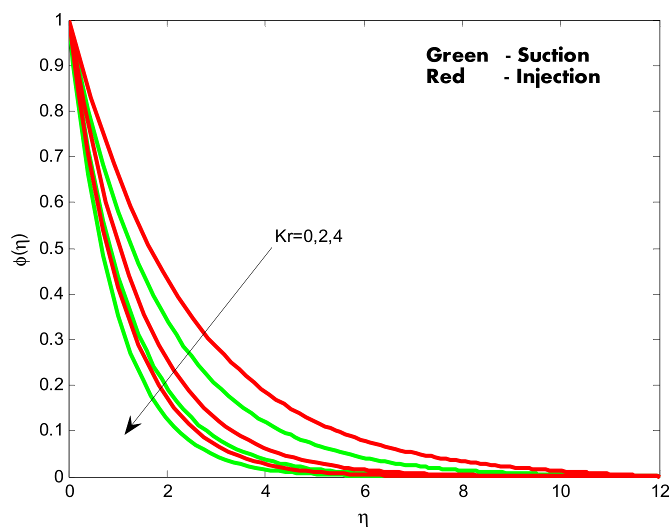

Fig. 9. Concentration distribution for different values of chemical reaction parameter *Kr*.

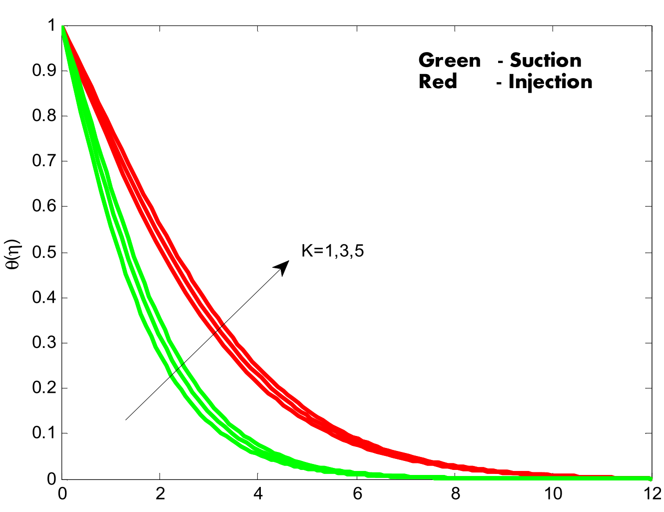

Fig. 7. Temperature distribution for different values of micropolar parameter *K*.

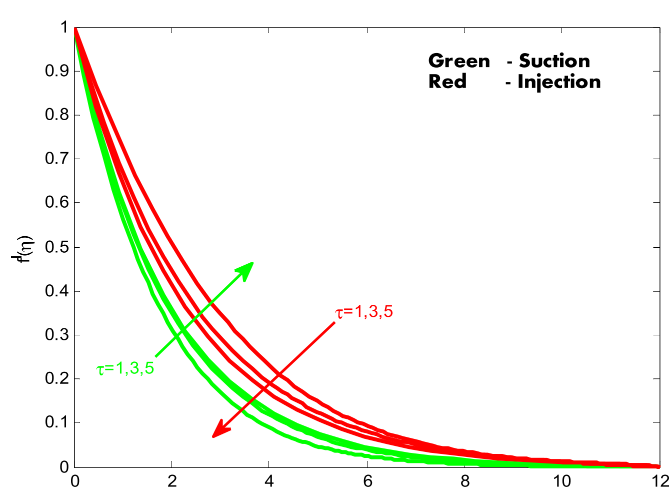

Fig. 10. Velocity distribution for different values of unsteadiness parameter τ.

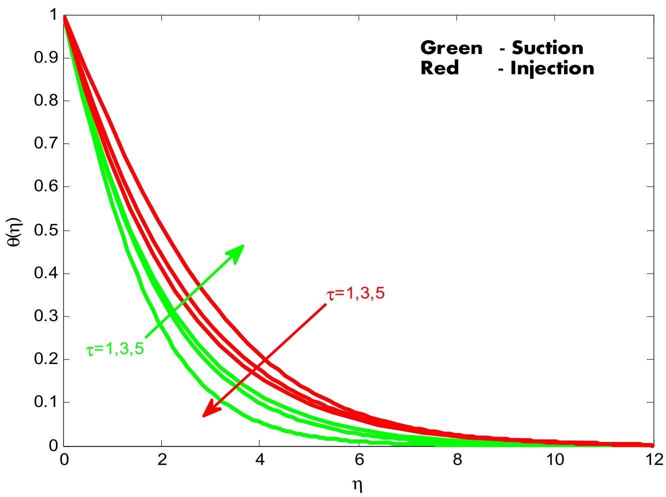

Fig. 11. Temperature distribution for different values of unsteadiness parameter τ.

parameter enhances the velocity, thermal boundary layer thicknesses. The influence of chemical reaction parameter on velocity and concentration profiles is displayed in Figs. 8 and 9. It is noticed from the figures that an increase in chemical reaction parameter causes to depreciate the velocity and concentration profiles. This agrees the general physical behavior of chemical reaction parameter that the increase in chemical reaction reduces the concentration and velocity boundary layer thickness.

Figs. 10 and 11 present the influence of unsteady parameter on velocity and temperature profiles respectively for both suction and injection cases. We noticed an interesting result that the enhancement in the value of unsteadiness parameter increases the velocity and temperature profiles in suction case. In injection case it takes reverse action and shown fall in velocity and temperature profiles. This agrees with the result obtained by Gupta et al. [10]. Physically an increase in the unsteadiness parameter increases the heat loss by the sheet in injection case. Due to this reason we seen fall in temperature profiles in injection case.

Figs. 12–15 display the influence of non-uniform heat source or sink on velocity and temperature profiles respectively for both suction and injection cases. It is evident from the figures that an

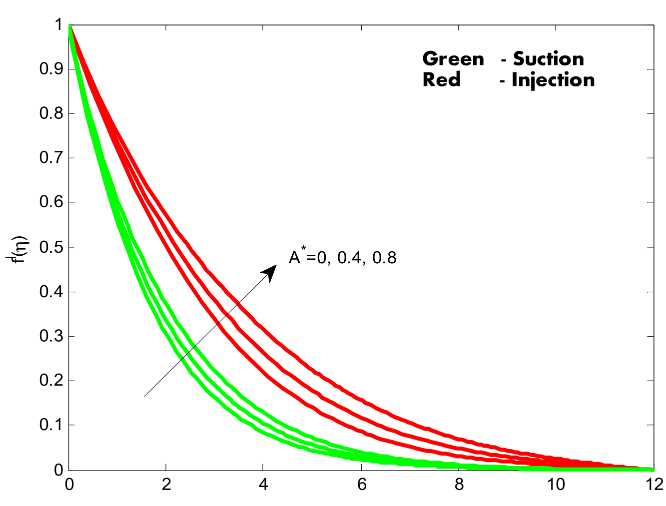

Fig. 12. Velocity distribution for different values of non-uniform heat source/sink parameter A^{*}.

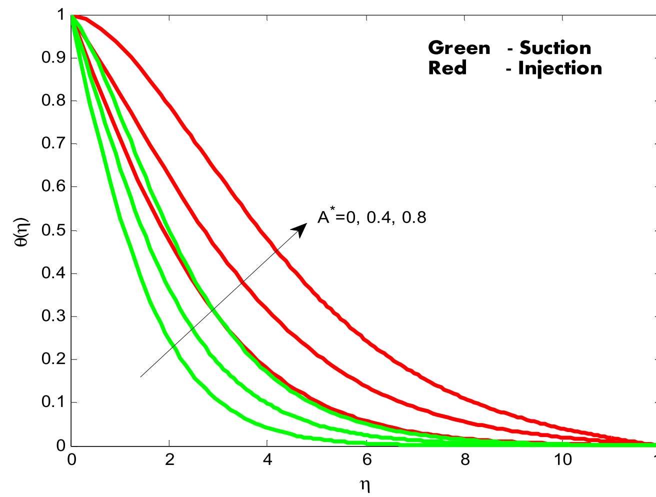

Fig. 13. Temperature distribution for different values of non-uniform heat source/sink parameter A^{*}.

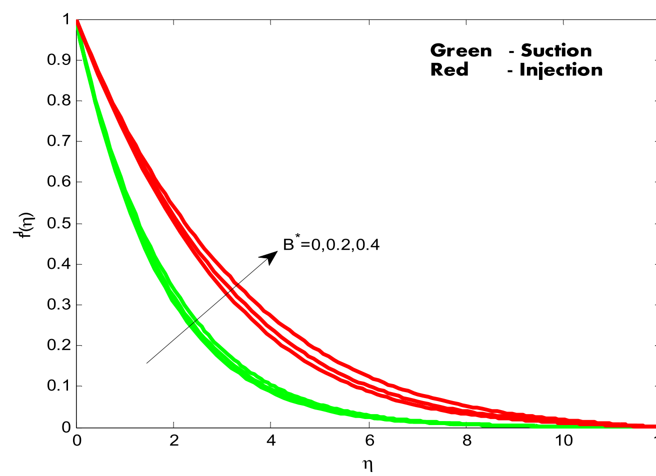

Fig. 14. Velocity distribution for different values of non-uniform heat source/sink parameter B^{*}.

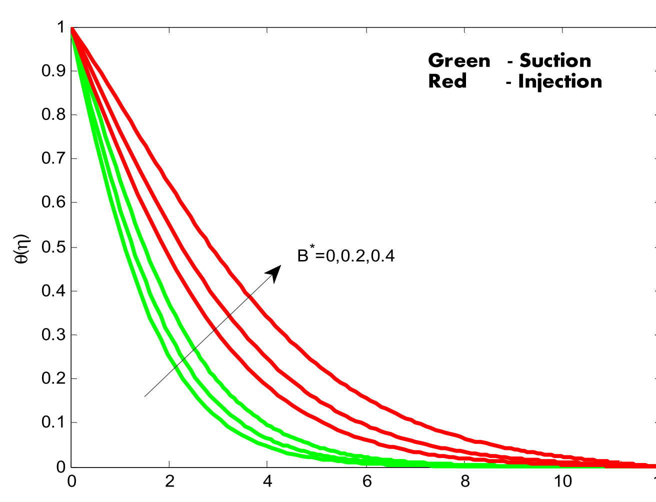

Fig. 15. Temperature distribution for different values of non-uniform heat source/sink parameter B^{*}.

Table 1
Comparison of the reduced Nusselt number obtained by the FEM [10] with $M = Kr = A^* = B^* = \delta_1 = 0$, $K = 2$ and $Pr = 0.733$ in shrinking case.

f_w	$\tau = 1,\ \delta = 3,\ Ec = 0.75,\ -\theta'(0)$	Present study	τ	$f_w = 3,\ \delta = 3,\ Ec = 0.75,\ -\theta'(0)$	Present study	δ	$f_w = 3,\ \tau = 1,\ Ec = 0.75,\ -\theta'(0)$	Present study
3	−0.24924	−0.249251	1	−0.24924	−0.249251	0	0.88895	0.888954
3.5	0.12914	0.129153	2	0.14660	0.146607	1	0.54444	0.544444
4	0.45990	0.459915	3	0.46597	0.465974	2	0.17095	0.170953
4.5	0.75067	0.750681	4	0.71556	0.715565	3	−0.24924	−0.249244
5	1.01279	1.012798	5	0.91656	0.916564	4	−0.73308	−0.733083

increase in non-uniform heat source or sink parameters enhances the velocity and temperature profiles. This agrees the general physical behavior of heat source/sink that the positive values of A^* and B^* acts like heat generators and negative values is for heat absorbers. Generally, increase in heat source develops the thermal and velocity boundary layer thickness. Table 1 displays the comparison of the present results with the existed results of Gupta et al. [10]. Present results showed an excellent agreement with the existed results. This indicates the validity of the present results. Table 2 shows the influence of various non-dimensional governing parameters on skin friction coefficient, Nusselt and Sherwood numbers. It is evident from the table that increase in the microploar parameter enhances the friction factor along with Sherwood number for both suction and injection cases but it reduces the heat transfer rate. An increase in the magneticfield parameter depreciates the skin friction coefficient, heat and mass transfer rate. The enhancement in the unsteadiness parameter decreases the friction factor and mass transfer rate. But it helps to enhance the heat transfer rate. A raise in the value of chemical reaction parameter reduces the friction factor and Nusselt number, improves the mass transfer rate. An increase in the non-uniform heat source/sink declines the heat transfer rate and enhances the friction factor and mass transfer rate.

Table 2
Variation in $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for different values of M, K, τ, Kr, A^*, B^* for suction and injection cases.

f_w	K	M	τ	Kr	A^*/B^*	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	1	1	1	0.2	0.1	−0.482185	0.410466	0.533872
	2	1	1	0.2	0.1	−0.456953	0.372160	0.536254
	3	1	1	0.2	0.1	−0.434805	0.339191	0.538322
−0.5	1	1	1	0.2	0.1	−0.356047	0.308058	0.467315
	2	1	1	0.2	0.1	−0.346130	0.286934	0.468490
	3	1	1	0.2	0.1	−0.335726	0.268082	0.469583
0.5	1	1	1	0.2	0.1	−0.482185	0.410466	0.533872
	1	2	1	0.2	0.1	−0.699710	0.174148	0.519895
	1	3	1	0.2	0.1	−0.893532	0.033011	0.508471
−0.5	1	1	1	0.2	0.1	−0.356047	0.308058	0.467315
	1	2	1	0.2	0.1	−0.552872	0.104524	0.454323
	1	3	1	0.2	0.1	−0.730850	0.077825	0.443523
0.5	1	1	1	0.2	0.1	−0.482185	0.410466	0.533872
	1	1	3	0.2	0.1	−0.490774	0.443244	0.520035
	1	1	5	0.2	0.1	−0.494258	0.460322	0.514164
−0.5	1	1	1	0.2	0.1	−0.356047	0.308058	0.467315
	1	1	3	0.2	0.1	−0.427943	0.392838	0.475262
	1	1	5	0.2	0.1	−0.452314	0.425025	0.481086
0.5	1	1	1	0	0.1	−0.478349	0.412062	0.501703
	1	1	1	0.5	0.1	−0.487347	0.408311	0.579045
	1	1	1	1	0.1	−0.494760	0.405213	0.647932
−0.5	1	1	1	0	0.1	−0.353280	0.308870	0.436856
	1	1	1	0.5	0.1	−0.359820	0.306946	0.510346
	1	1	1	1	0.1	−0.365328	0.305316	0.576443
0.5	1	1	1	0.2	0	−0.487494	0.456865	0.533000
	1	1	1	0.2	0.4	−0.465664	0.268815	0.536611
	1	1	1	0.2	0.8	−0.442099	0.073611	0.540562
−0.5	1	1	1	0.2	0	−0.359712	0.347226	0.466612
	1	1	1	0.2	0.4	−0.344776	0.189280	0.469498
	1	1	1	0.2	0.8	−0.329041	0.027707	0.472587

5. Conclusions

This study presents a numerical investigation of the influence of non-uniform heat source/sink, mass transfer and chemical reaction on unsteady mixed convection boundary layer flow of a magneto-micropolar fluid past a stretching/shrinking sheet in the presence of viscous dissipation and suction/injection. The conclusions of the present

- Positive values of non-uniform heat source/sink parameters acts like heat generators and these parameters helps to enhance the mass transfer rate along with microrotation.
- Micropolar parameter have tendency to increase the velocity and temperature boundary layers.
- A raise in the values of Magneticfield parameter and Chemical reaction parameter enhances the friction factor and mass transfer rate and depreciates the Nusselt number.
- An increase in unstedainess parameter causes to declines the velocity, temperature and concentration profiles in injection case. But enhances the heat transfer rate.
- It is also found that an increase in suction parameter and viscous dissipation parameter helps to fast rate of cooling.

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Nomenclature

u, v	velocity components in x and y directions
c_p	specific heat capacity at constant pressure
f	dimensionless velocity
h	dimensionless microrotation
g_e	gravitational acceleration
j	microinertia density
K	micropolar parameter
N	microrotation component
S	constant characteristic of the fluid
w_i	weight functions
x	distance along the surface
y	distance normal to the surface
U_w	velocity of the sheet
V_w	velocity of the wall
t	time
T_w	temperature of the fluid
T_∞	free stream temperature
C_w	concentration of the fluid

C_∞	free stream Concentration
q	heat flux
B	magneticfield strength
D_m	molecular diffusivity
C_f	skin friction coefficient
Nu_x	local Nusselt number
Sh_x	local Sherwood number
Re_x	local Reynolds number
Pr	Prandtl number
Sc	Schmidt number
Ec	Eckert number
f_w	suction/injection parameter

Greek symbols

ρ	fluid density
k	thermal conductivity
β	coefficient of volume expansion due to temperature
β^*	coefficient of volume expansion due to concentration
σ	electrical conductivity
γ	spin gradient viscosity
η	similarity variable
μ	dynamic viscosity
ν	kinematic viscosity
θ	dimensionless temperature
ϕ	dimensionless concentration
ψ	stream function
δ, δ_1	buoyancy parameters
τ	unsteadiness parameter
τ_w	wall shear stress

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